

Foundations of Probabilistic Proofs

A course by **Alessandro Chiesa**

Lecture 25

Parallel Repetition



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Parallel Repetition

BASIC QUESTION: How to reduce the soundness error of a probabilistic proof?

BASIC ANSWER: Run the probabilistic proof t times in sequence.

A straightforward analysis shows that the soundness error decreases from ϵ to ϵ^t .

PROBLEM: Efficiency measures increase by a multiplicative factor of t .

PARALLEL REPETITION loosely refers to ideas for reducing the soundness error of a probabilistic proof while **PRESERVING** certain efficiency measures.

Parallel repetition is a **TRANSFORMATION** separately defined for each proof model:

- PR_{IP} for IPs (interactive proofs)
- PR_{MIP} for MIPs (multi-prover interactive proofs)
- PR_{PCP} for PCPs (probabilistically checkable proofs)
- PR_{IOP} for IOPs (interactive oracle proofs)

While there are similarities and connections across models, parallel repetition behaves surprisingly different in each case.

Parallel Repetition for IPs

Let (P, V) be an IP. Fix a repetition parameter t . Define $(P_t, V_t) := \text{PR}_{\text{IP}}(P, V, t)$ as follows:

$P_t(x)$

Sample prover randomness $\sigma_1, \dots, \sigma_t \leftarrow \{0, 1\}^{pr}$

For $i \in [t]$: $a_{i,j} := P(b_{i,j-1}; \sigma_i)$

For $j=1, \dots, k$:

$\xrightarrow{(a_{i,j})_{i \in [t]}}$

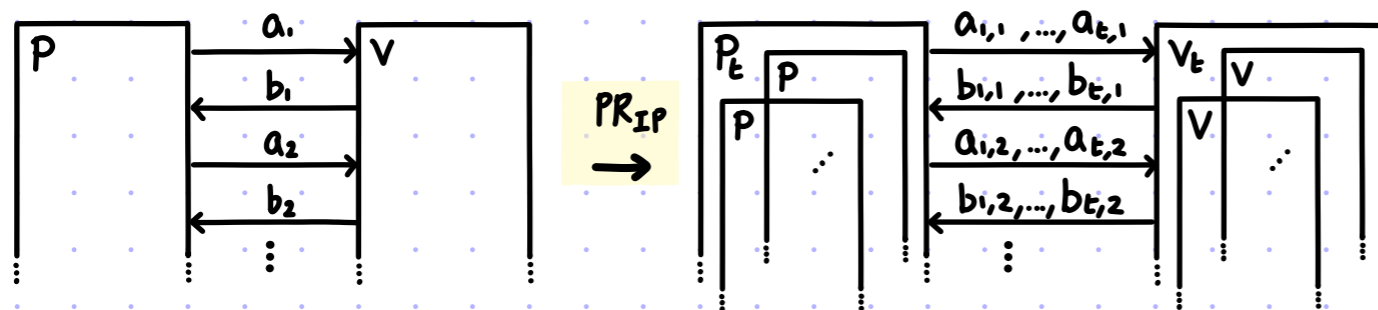
$\xleftarrow{(b_{i,j})_{i \in [t]}}$

$V_t(x)$

Sample verifier randomness $\xi_1, \dots, \xi_t \leftarrow \{0, 1\}^{vr}$

For $i \in [t]$: $b_{i,j} := V(a_{i,j}; \xi_i)$

Check that all t executions of V accept.



- round complexity: $k \mapsto k$
- communication complexity: $(pc, vc) \mapsto (pc', vc') = (t \cdot pc, t \cdot vc)$
- completeness error: $\epsilon_c \mapsto \epsilon_c' = 1 - (1 - \epsilon_c)^t \leq t \cdot \epsilon_c$ (completeness error increases slightly)
- soundness error: $\epsilon_s \mapsto \epsilon_s' = \epsilon_s^t$ ← This seems intuitive but let's prove it.

Multi-Prover Interactive Proofs

A **multi-prover interactive proof (MIP)** is a probabilistic proof where a single verifier interacts with multiple **NON-COMMUNICATING** (possibly malicious) provers.

We say that (P, V) is an **MIP system** for a language L with p provers, completeness error ϵ_c , and soundness error ϵ_s if the following holds:

① **COMPLETENESS:** $\forall x \in L \quad \Pr_{\alpha, (\sigma_i)_{i \in [p]}, \beta} \left[\langle (P(i, x, \alpha; \sigma_i))_{i \in [p]}, V(x; \beta) \rangle = 1 \right] \geq 1 - \epsilon_c.$

② **SOUNDNESS:** $\forall x \notin L \quad \forall (\tilde{P}_i)_{i \in [p]} \quad \Pr_{\beta} \left[\langle (\tilde{P}_i)_{i \in [p]}, V(x; \beta) \rangle = 1 \right] \leq \epsilon_s.$

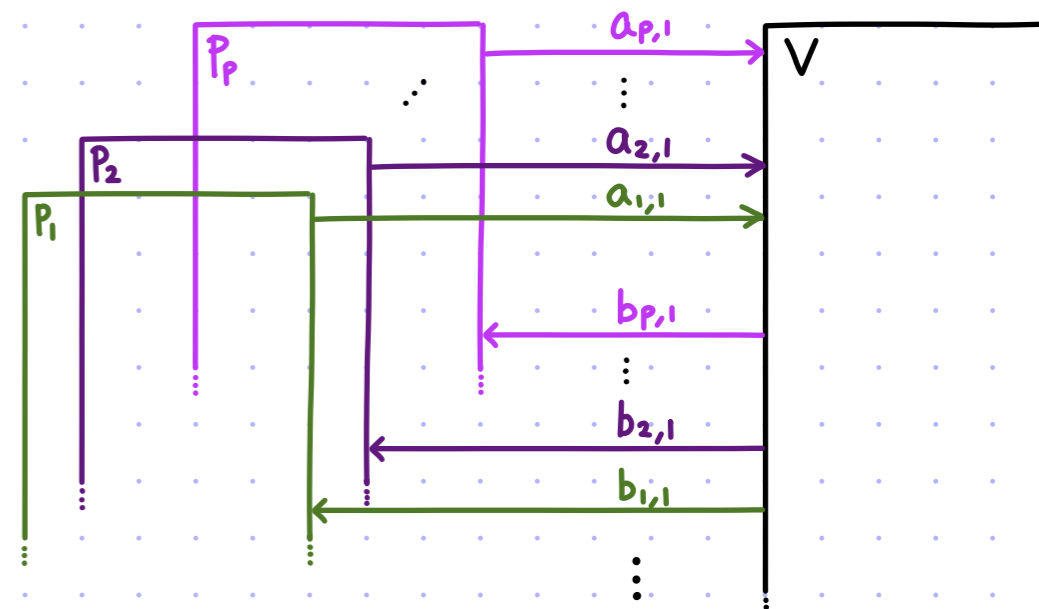
Each round $j \in [K]$ of interaction works as follows:

$\forall i \in [p]$ prover i sends a message $a_{i,j}$ to the verifier,

$\forall i \in [p]$ the verifier sends a message $b_{i,j}$ to prover i .

Efficiency measures:

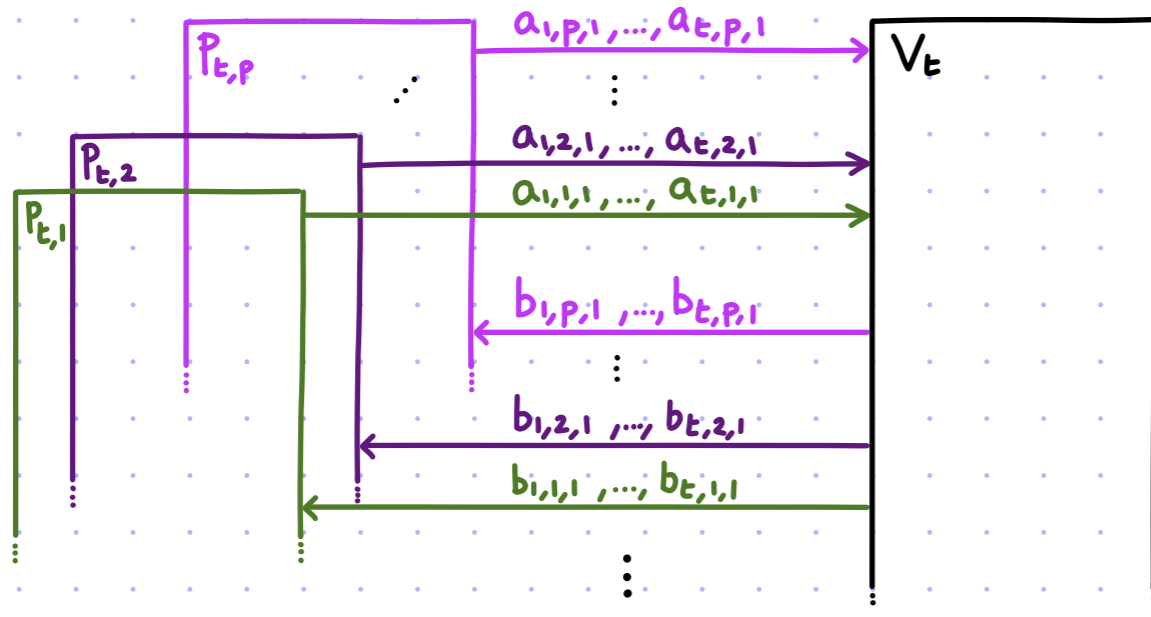
- p : number of provers
 - K : round complexity
 - pc : (total) prover-to-verifier communication
 - vc : (total) verifier-to-prover communication
- could also separate by prover and round



Parallel Repetition for MIPs

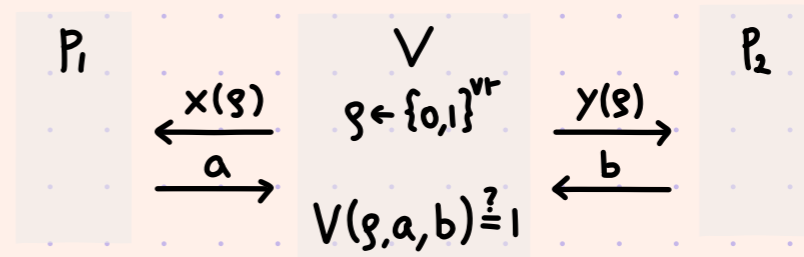
Let (P, V) be an MIP. Fix a repetition parameter t .

Define $(P_t, V_t) := \text{PR}_{\text{MIP}}(P, V, t)$ as follows:

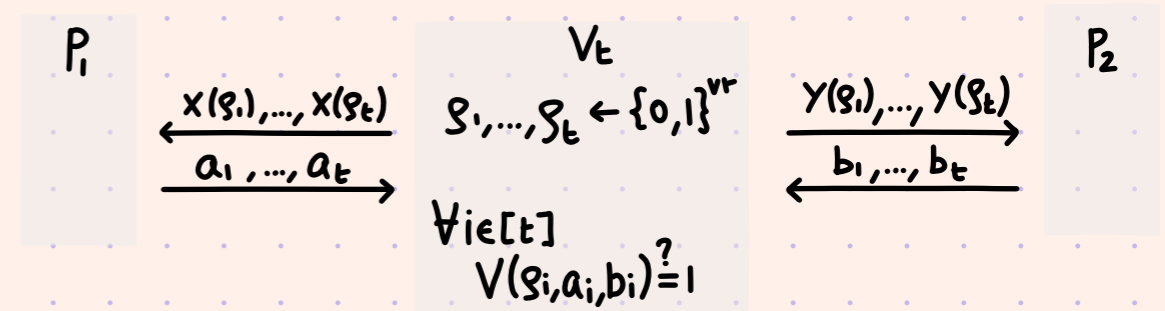


Notable special case:

MIP with 2 provers, 1 round (verifier first)



Its t -wise PR has the same format:



- number of provers: $p \mapsto p$
- round complexity: $k \mapsto k$
- communication complexity: $(pc, vc) \mapsto (pc', vc') = (t \cdot pc, t \cdot vc)$
- completeness error: $\epsilon_c \mapsto \epsilon_c' = 1 - (1 - \epsilon_c)^t \leq t \cdot \epsilon_c$ (completeness error increases slightly)
- soundness error: $\epsilon_s \mapsto \epsilon_s' = ?$

claim: $\epsilon_s^t \leq \epsilon_s' \leq \epsilon_s$

Q: $\epsilon_s' = \epsilon_s^t$?

best prover in each repetition

winning all repetitions at least as hard as winning one

Refuting Expectation

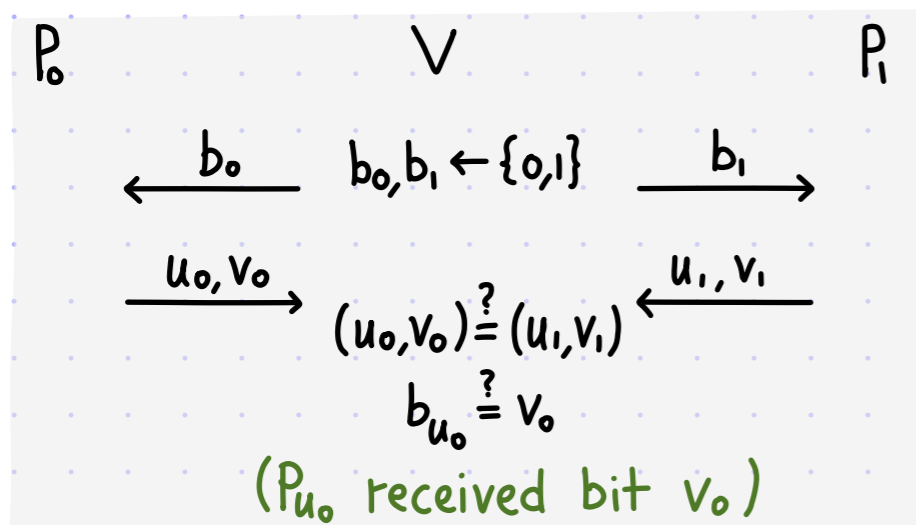
[1/2]

CONJECTURE [Fortnow, Rompel, Sipser 1988]: PR for MIPs reduces soundness error $\epsilon \mapsto \epsilon^k$.

[Fortnow 1989]: counterexample to conjecture.

Here we see a simpler counterexample from [Feige 1991]:

MIP for **NON-INTERACTIVE AGREEMENT** (which has $p=2$ provers and $k=1$ rounds).



claim: $\epsilon = 1/2$ where $\epsilon := \max_{\tilde{P}_0, \tilde{P}_1} \Pr [\langle (\tilde{P}_0, \tilde{P}_1), V \rangle = 1]$

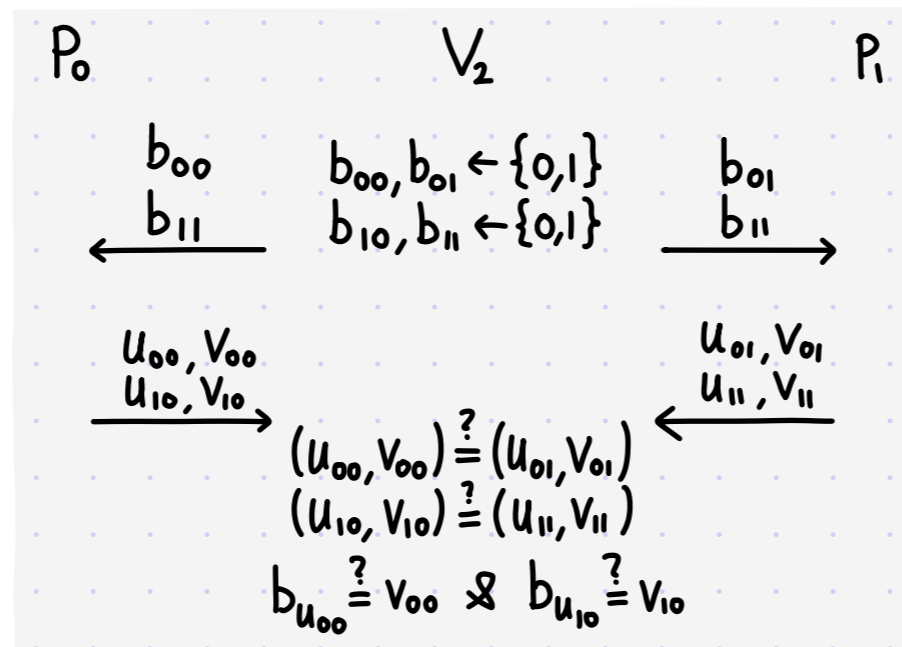
proof:

- $\epsilon \geq 1/2$: P_0 answers $(0, b_0)$ and P_1 answers $(0, \text{random bit})$
- $\epsilon \leq 1/2$: wlog \tilde{P}_0, \tilde{P}_1 agree to guess P_0 's bit b_0 ;
hence \tilde{P}_1 must guess b_0 but has no information about b_0 . ■

Refuting Expectation

[2/2]

Consider the 2-wise PR of the MIP for non-interactive agreement:



claim: $\varepsilon_2 = 1/2$ where $\varepsilon_2 := \max_{\tilde{P}_0, \tilde{P}_1} \Pr [\langle (\tilde{P}_0, \tilde{P}_1), V_2 \rangle = 1]$

proof:

- $\varepsilon_2 \leq 1/2$: $\varepsilon_2 \leq \varepsilon = 1/2$
- $\varepsilon_2 \geq 1/2$: \tilde{P}_0 sends $\begin{matrix} 0, b_{00} \\ 1, b_{00} \end{matrix}$ and \tilde{P}_1 sends $\begin{matrix} 0, b_{11} \\ 1, b_{11} \end{matrix}$.

If $b_{00} = b_{11}$ then \tilde{P}_0 and \tilde{P}_1 win BOTH iterations. ■

Another view: $\Pr [\text{win1} \wedge \text{win2}] = \Pr [\text{win1}] \cdot \Pr [\text{win2} | \text{win1}] = \frac{1}{2} \cdot 1 = \frac{1}{2}$.

In the second iteration, conditioning creates implicit communication between provers.

Verbitsky's Theorem

The counterexample shows that $\epsilon_t \neq \epsilon^t$, even in the minimal setting of $p=2$ provers and $k=1$ rounds.

But PR for the counterexample still "works", just slower than expected.

lemma [Feige 1991]: For the MIP for non-interactive agreement, $\epsilon_t \leq \left(\frac{1}{2}\right)^{t/2} = \left(\frac{1}{\sqrt{2}}\right)^t$.

More precisely:

- t even $\rightarrow \epsilon_t = \left(\frac{1}{2}\right)^{t/2}$
- t odd $\rightarrow \left(\frac{1}{2}\right)^{\frac{t+1}{2}} < \epsilon_t < \left(\frac{1}{2}\right)^{t/2}$

Verbitsky proved that PR for 1-round MIPs always works "in the limit":

theorem: Let ϵ be the soundness error of a 1-round MIP verifier V , and ϵ_t the soundness error of its t -wise parallel repetition V_t .

For every instance $x \notin L$, if $\epsilon(x) < 1$ then $\lim_{t \rightarrow \infty} \epsilon_t(x) = 0$.
(i.e. $\forall \epsilon_0 \exists t_0 \forall t > t_0 \epsilon_t(x) < \epsilon_0$)

Towards The Parallel Repetition Conjecture

Oleg Verbitsky
Moscow State University



Verbitsky proved the theorem for $p=2$ provers.

The proof approach extends to work for any p .

study of combinatorial objects
where "order" appears if large enough

The proof is a direct application of a deep result in **RAMSEY THEORY**, and only shows that the soundness error ϵ_t decreases **VERY SLOWLY**.

A Result On Combinatorial Lines

Let A be a finite alphabet and $\Delta \notin A$ a special symbol.

A word is a string in A^* and a root is a string in $(A \cup \{\Delta\})^* \setminus A^*$.

For a root r_t and $a \in A$, $r_t(a)$ is the word (in A^*) obtained by replacing each Δ with a .

Eg: $A = \{1, 2, 3\}$ $r_t = 31\Delta 12\Delta$ $r_t(1) = 311121$ $r_t(3) = 313123$

A combinatorial line in A^t is a subset $L \subseteq A^t$ of the form $\{r_t(a)\}_{a \in A}$ for a root r_t .

Eg: $A = \{1, 2, 3\}$ $r_t = 31\Delta 12\Delta$ $L_{r_t} = \begin{pmatrix} 3 & 1 & 1 & 1 & 2 & 1 \\ 3 & 1 & 2 & 1 & 2 & 2 \\ 3 & 1 & 3 & 1 & 2 & 3 \end{pmatrix}$

Combinatorial lines are in correspondence with roots, of which there are $(|A|+1)^t - |A|^t$.

Define $N(A, t) := \max \{ |W| \mid W \subseteq A^t \text{ contains NO combinatorial lines} \} \in \{0, 1, \dots, |A|^t\}$.

theorem [Furstenberg & Katznelson 1991]: $\forall A \forall \varepsilon > 0 \exists T \forall t > T \frac{N(A, t)}{|A|^t} < \varepsilon$

This is a density version of the Hales–Jewett theorem $\left(\begin{matrix} \forall A \forall r \exists T \forall t \geq T \\ \text{every } r\text{-coloring of } A^t \\ \text{has a monochromatic line} \end{matrix} \right)$.

In 2010 the Polymath project gave a quantitative bound: $T \sim \text{Ack}_{|A|} (1/\varepsilon)$.

Ackermann function:

$\text{Ack}_m(1) = 2, \text{Ack}_1(n) = 2n, \text{Ack}_m(n) = \text{Ack}_{m-1}(\text{Ack}_m(n-1))$

Raz's Theorem

Much better rate of decay is known for a minimal special case:

A Parallel Repetition Theorem

Ran Raz
Weizmann Institute



theorem: [Raz 1995] There exists $c > 0$ s.t. the following holds.

Let (P, V) be an MIP for a language L with soundness error ϵ , $p=2$ provers, and $k=1$ rounds.
Then $\forall x \notin L \quad \epsilon(x) \leq 1 - \delta \rightarrow \epsilon_t(x) \leq (1 - \delta^c)^{\Omega(\frac{t}{\log |\Sigma|})}$, where Σ is the alphabet for prover answers.

- [Feige Verbitsky 1996]: the dependence on $\log |\Sigma|$ is necessary
- [Holenstein 2010]: $c \leq 3$ (vs. $c \leq 32$ in Raz's proof)
- cannot expect $c \leq 1$ in general (strong parallel repetition is the study of when $c \approx 1$)

For $p=3$ provers and $k=1$ rounds some recent progress on rate of decay (in special cases).

Understanding $p > 2$ provers or $k > 1$ rounds remains a **CHALLENGING OPEN PROBLEM**.

Main Lemma Behind Raz's Theorem

Fix strategies \tilde{P}_1, \tilde{P}_2 against the t -wise parallel repeated MIP verifier $V_t(x)$.

For $i \in [t]$, $W_i := V(x, g_i, \tilde{P}_1(x(g_1), \dots, x(g_t))[i], \tilde{P}_2(y(g_1), \dots, y(g_t))[i])$. For $S \subseteq [t]$, $W_S := \bigwedge_{i \in S} W_i$.

By assumption, $\forall i \in [t] \Pr[W_i] \leq 1 - \delta$.

GOAL: upper bound $\Pr[\bigwedge_{i \in [t]} W_i]$.

Main Lemma: $\exists \gamma$ (that depends on $V(x)$) $\forall S \subseteq [t]$ with $|S| \leq \gamma \cdot t$ if $\Pr[W_S] \geq 2^{-\gamma \cdot t}$ then $\exists i \in [t] \setminus S$ s.t. $\Pr[W_i | W_S] \leq 1 - \frac{\delta}{2}$.

This implies the theorem as explained below.

↑ proved via sophisticated analysis based on Information Theory

Initialize $S := \emptyset$ and do the following while $|S| \leq \gamma \cdot t$:

- ① If $\Pr[W_S] < 2^{-\gamma \cdot t}$ then exit loop.
- ② If $\Pr[W_S] \geq 2^{-\gamma \cdot t}$ then add to S some $i \in [t] \setminus S$ s.t. $\Pr[W_i | W_S] \leq 1 - \frac{\delta}{2}$.

If the first condition is met at some iteration then $\Pr[\bigwedge_{i \in [t]} W_i] \leq \Pr[W_S] < 2^{-\gamma \cdot t}$.

If the first condition is never met, then we obtain $S = \{i_1, i_2, \dots, i_{\gamma \cdot t}\}$ s.t.

$$\Pr[\bigwedge_{i \in [t]} W_i] \leq \Pr[W_S] = \Pr[W_{i_1}] \cdot \Pr[W_{i_2} | W_{\{i_1\}}] \cdot \Pr[W_{i_3} | W_{\{i_1, i_2\}}] \cdot \dots \leq (1 - \frac{\delta}{2})^{\gamma \cdot t}.$$

We conclude that $\Pr[\bigwedge_{i \in [t]} W_i] \leq \max\{2^{-\gamma \cdot t}, (1 - \frac{\delta}{2})^{\gamma \cdot t}\}$.

Repetition Applied to the PCP Theorem

Recall the PCP Theorem: $NP \subseteq PCP[\epsilon_c = 0, \epsilon_s = 1/2, \Sigma = \{0,1\}, \ell = \text{poly}(n), q = O(1), r = O(\log n)]$.

Rerunning the PCP verifier gives arbitrarily small ϵ_s with large-enough q :

corollary: $\forall \epsilon_s > 0 \exists q$ s.t. $NP \subseteq PCP[\epsilon_c = 0, \epsilon_s, \Sigma = \{0,1\}, \ell = \text{poly}(n), q, r = O(\log n)]$

Repetition via PR of MIPs gives arbitrarily small ϵ_s with large-enough Σ (and with $q=2$):

theorem: $\forall \epsilon_s > 0 \ NP \subseteq PCP[\epsilon_c = 0, \epsilon_s, \Sigma = \{0,1\}^{O(\log \frac{1}{\epsilon_s})}, \ell = n^{O(\log \frac{1}{\epsilon_s})}, q=2, r = O(\log \frac{1}{\epsilon_s} \cdot \log n)]$

proof: $PCP \xrightarrow{\textcircled{1}} MIP \xrightarrow{\textcircled{2}} \text{repeated MIP} \xrightarrow{\textcircled{3}} PCP$

① From PCP to 2-prover 1-round MIP (via trivial query bundling)

$$PCP[\epsilon_c, \epsilon_s, \Sigma, \ell, q, r] \subseteq MIP[\epsilon_c, \epsilon_s = 1 - \frac{1-\epsilon_s}{q}, p=2, k=1, (\Sigma_v, \Sigma_p) = ([\ell]^q, \Sigma^q), r' = r + \log q]$$

$$\text{yields } NP \subseteq MIP[\epsilon_c = 0, \epsilon_s = O(1), p=2, k=1, (\Sigma_v, \Sigma_p) = (\{0,1\}^{O(\log n)}, \{0,1\}^{O(1)}), r = O(\log n)]$$

② Apply PR for MIPs: $NP \subseteq MIP[\epsilon_c = 0, \epsilon_s = \epsilon_t, p=2, k=1, (\Sigma_v, \Sigma_p) = (\{0,1\}^{O(t \cdot \log n)}, \{0,1\}^{O(t)}), r = O(t \cdot \log n)]$

③ Evaluate the MIP as a PCP: $NP \subseteq PCP[\epsilon_c = 0, \epsilon_s = \epsilon_t, \Sigma = \{0,1\}^{O(t)}, \ell = n^{O(t)}, q=2, r = O(t \cdot \log n)]$

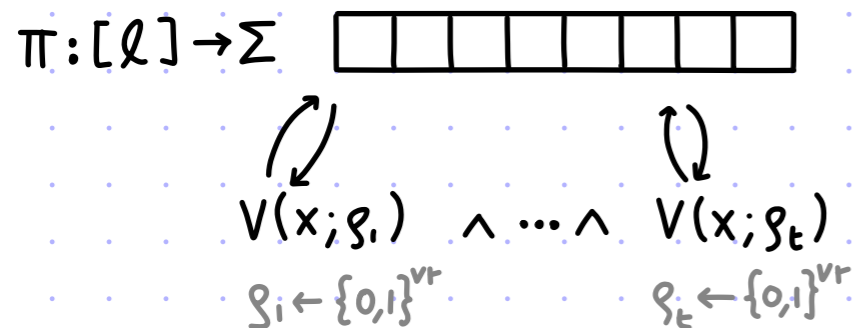
By Raz's Theorem, $\epsilon_t = \exp(-t)$, so can set $t = O(\log \frac{1}{\epsilon_s})$. ■

The **main limitation** of parallel repetition is that if we want $\ell = \text{poly}(n)$ then $\epsilon_s = \Omega(1)$.

Reducing Soundness Error for PCPs

Q: How to reduce the soundness error of a PCP?

SIMPLE: repeat the PCP verifier multiple times



If the honest PCP prover uses randomness (e.g. for zk) one may have to sample t PCP strings π_1, \dots, π_t (one per repetition)

For every $t \in \mathbb{N}$, the t -wise repetition does the following:

- $\Sigma \mapsto \Sigma' = \Sigma$ alphabet does not change
 - $l \mapsto l' = l$ proof length does not change
 - $q \mapsto q' = t \cdot q$ query complexity increases
 - $vr \mapsto vr' = t \cdot vr$ randomness complexity increases
 - $\epsilon_c \mapsto \epsilon_c' = 1 - (1 - \epsilon_c)^t \leq t \cdot \epsilon_c$ completeness error increases
 - $\epsilon_s \mapsto \epsilon_s' = \epsilon_s^t$ soundness error decreases exponentially
- randomness-efficient error reduction (e.g. via expanders) has no better query complexity

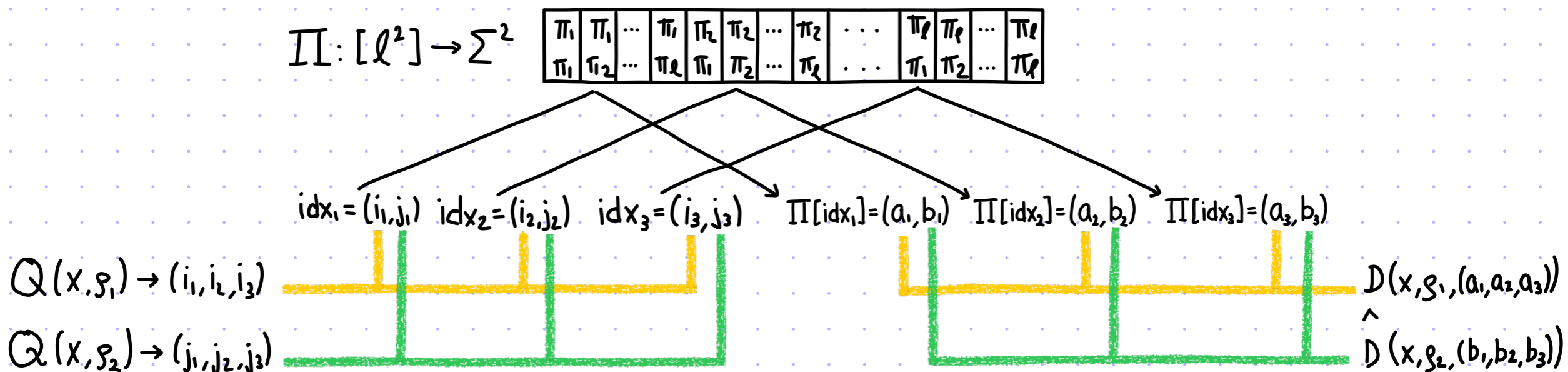
How to reduce soundness error while preserving query complexity?

Parallel Repetition for PCPs

[1/2]

IDEA: bundle queries across multiple repetitions ("BUNDLED REPETITION")

Example with $t=2$ repetitions for a PCP with $q=3$ queries.



The query complexity did NOT change.

Each query consists of two indices and is answered via two symbols.

→ The proof length and alphabet size square.

Parallel Repetition for PCPs

[2/2]

The t -wise parallel repetition of a (non-adaptive) PCP:

$P_t(x)$

1. Compute $\pi := P(x) \in \Sigma^l$.
2. Set $\Pi := ((\pi[i_1], \dots, \pi[i_t]))_{i_1, \dots, i_t \in [l]}$.
3. Output $\Pi: [l^t] \rightarrow \Sigma^t$.

$V_t(x)$

1. Sample $s_1, \dots, s_t \in \{0, 1\}^r$.
2. Deduce query sets: $\forall i \in [t], Q_i := Q(x, s_i) \subseteq [l]$.
3. Construct tuples: $\forall j \in [q] \text{ idx}_j := (Q_1[j], \dots, Q_t[j])$.
4. Check that $\bigwedge_{i \in [t]} D(x, s_i, \Pi[\text{idx}_1]_i \dots \Pi[\text{idx}_q]_i) = 1$.

- $\Sigma \mapsto \Sigma' = \Sigma^t$ alphabet increases exponentially
- $l \mapsto l' = l^t$ proof length increases exponentially
- $q \mapsto q' = q$ query complexity does NOT increase
- $r \mapsto r' = t \cdot r$ randomness complexity increases
- $\epsilon_c \mapsto \epsilon_c' = 1 - (1 - \epsilon_c)^t \leq t \cdot \epsilon_c$ completeness error increases slightly
- $\epsilon_s \mapsto \epsilon_s' = ?$ How does PR of a PCP affect soundness error?

Refuting Expectation, Again

Parallel repetition for PCPs **fails to work**.

theorem: \exists 2-query PCP for NP-complete language L with soundness error ϵ s.t.

$\forall x \notin L \quad \epsilon(x) < 1$ and $\lim_{t \rightarrow \infty} \epsilon_t(x) = 1$ (In fact, for infinitely many $x \notin L$, $\epsilon_{t+1}(x) > \epsilon_t(x) \forall t \in \mathbb{N}$.)

In particular, **NOT** true that $\epsilon(x)^t \leq \epsilon_t(x) \leq \epsilon(x)$.

Here is a **CRITERION** of when PR for PCPs works.

def: The **MIP projection** of a PCP verifier V is the MIP verifier V_{MIP} that works as follows:

- $V_{\text{MIP}}(x)$:
1. Sample PCP randomness $g \leftarrow \{0,1\}^r$.
 2. Deduce query set $Q := V_q(x, g) \subseteq [l]$.
 3. For every $i \in [q]$, send $Q[i]$ to prover i , and get response $a_i \in \Sigma$.
 4. Check that $V_D(x, g, (a_i)_{i \in [q]}) = 1$.

lemma: Let (P, V) be a PCP for a language L with soundness error ϵ . Let ϵ_t be the soundness error of its t -wise parallel repetition. Let V_{MIP} be the MIP projection V , with soundness error ϵ_{MIP} .

Then $\forall x \notin L \quad \lim_{t \rightarrow \infty} \epsilon_t(x) = 0 \iff \epsilon_{\text{MIP}}(x) < 1$.

Consistent Parallel Repetition

A simple variant of PR for PCPs does always work.

The t -wise consistent parallel repetition (CPR) of a (non-adaptive) PCP:

$P_t(x)$

1. Compute $\pi := P(x) \in \Sigma^l$.
2. Set $\Pi := ((\pi[i_1], \dots, \pi[i_t]))_{i_1, \dots, i_t \in [l]}$.
3. Output $\Pi: [l^t] \rightarrow \Sigma^t$.

$V_t(x)$

1. Sample $s_1, \dots, s_t \in \{0, 1\}^r$.
2. Deduce query sets: $\forall i \in [t], Q_i := Q(x, s_i) \subseteq [l]$.
3. Construct tuples: $\forall j \in [q] \text{ idx}_j := (Q_1[j], \dots, Q_t[j])$.
4. Check that $\bigwedge_{i \in [t]} D(x, s_i, \Pi[\text{idx}_1]_i \dots \Pi[\text{idx}_q]_i) = 1$.
5. If $\exists i, i' \in [t], j, j' \in [q]$ s.t.
 $Q_i[j] = Q_{i'}[j']$ and $\Pi[\text{idx}_j]_i \neq \Pi[\text{idx}_{j'}]_{i'}$, reject.

theorem: Let (P, V) be a PCP for a language L with soundness error ϵ .

Let ϵ_t be the soundness error of its t -wise CONSISTENT parallel repetition.

Then $\forall x \notin L \quad \epsilon(x) < 1 \rightarrow \epsilon_t(x) < \binom{2^r}{\epsilon(x) \cdot 2^r} \cdot \epsilon(x)^t$ (in particular, $\lim_{t \rightarrow \infty} \epsilon_t(x) = 0$).

The proof is an elementary counting argument to upper bound the winning set of V_t .

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